

## Publications

- ISO/BIPM Guide: Uncertainty of measurement (Example resistor) (PDF)

*Author: Dr. Wolfgang Kessel, Braunschweig*

Starting from the general concept of a measuring process and the naive view of the measurement uncertainty, the paper presents the modern GUM view of the measurement uncertainty and discusses the interdependence of the standard measurement uncertainty and the expanded measurement uncertainty the last one being mostly used in trade and industry.

- How to treat correlation in the uncertainty budget, when combining results from different measurements (PDF)

*Authors: Rüdiger Kessel, Michael Berglund, Philip Taylor and Roger Wellum*

The ISO/BIPM Guide to the Expression of Uncertainty in Measurement (GUM) describes a method to evaluate the associated uncertainty of a measurement result. It is still an ongoing challenge to adapt the Guide to the different fields of metrology. In chemical analysis results from different measurements must often be combined. This paper will discuss cases where correlation can have an important influence on the uncertainty of the result. A scheme will be presented for the calculation of the correlation between results using the uncertainty budgets. Implementing a model, which includes the correlation, can significantly change the importance of some parameters. It also gives the analyst a better understanding of the major sources of uncertainties in the measurement process.

- A Novel Approach to Uncertainty Evaluation of Complex Measurements in Isotope Chemistry (PDF)

*Author: Rüdiger Kessel*

The class of chemical metrological (analytical) methods that can be summarised as isotope measurements is well understood and used for reference material characterisation and reference measurements. The aim of this thesis is to demonstrate that the methods given by the Guide to the Expression of Uncertainty in Measurement (GUM) can be used to evaluate the results of the class of chemical isotope measurements. The GUM methods were used before in this field on a case-by-case basis with the focus on individual measurements. The new scope of this thesis is to look at the evaluation techniques of the class as a whole. With this it has been possible to identify and to solve some common problems. Additionally, software tools were developed to improve and simplify the generation of the evaluation models. The influence of different evaluation models on the result and its uncertainty was studied. General evaluation models were developed that can be tailored to the specific measurement situation.

- Correlation in uncertainty of measurement - a discussion of state of the art techniques (PDF)

*Authors: Rüdiger Kessel and Raghu Kacker*

The Guide to the expression of uncertainty has been around for 15 years and has been widely adopted by science and industry. Over time more and more complex measurements are evaluated based on these principles. As a consequence the correlation between quantities has become an important issue in the evaluation of measurement uncertainty. In this paper we will give an overview about covariance and correlation and the different state of the art techniques to handle them during the uncertainty evaluation. We will discuss the handling of observations by extending the well known calculus for the degrees of freedom for correlated cases. Next we will discuss the difficulties and limitations in handling correlations with Monte Carlo simulations together with a practical algorithm to ensure that correlation matrixes are positive semi-definite.

- Framework for evaluation of uncertainty with test of linearity using covering arrays (PDF)

*Authors: Rüdiger Kessel and Raghu Kacker*

Since the Guide to the Expression of Uncertainty in Measurement (GUM) was published in 1993 it has changed the evaluation of physical and chemical measurements. Nowadays almost all high level measurements include a detailed evaluation of uncertainty. This allows the scientific community to do the next step and evaluate uncertainty for derived evaluations like parameter fittings. The evaluation of the uncertainty for complicated parameters like the results from non-linear fitting procedures can be carried out in two steps. The first step is a sensitivity analysis of the evaluation algorithm and a test of mutual independence of the parameters. If the fitting algorithm is sufficiently robust a linear model is derived from the fitting algorithm which is then used in a second step to evaluate the uncertainty of the fitting parameters. This paper discusses the sensitivity analysis in detail with the emphasis on possibilities to check for robustness and linearity. An efficient method based on covering arrays is presented to test for hidden couplings between the input parameters inside the evaluation model.

- Application of consistency checking to evaluation of uncertainty in multiple replicate measurements (PDF)

*Authors: Rüdiger Kessel, Michael Berglund and Roger Wellum*

Use of repeated measurements in quantitative chemical analysis is common but leads to the problem of how to combine the measurement values and produce a result with an uncertainty following the GUM. There is often confusion between repeated indications or observations of an input quantity, for whose uncertainty the GUM prescribes a type A evaluation, and complete measurements repeated on multiple sub-samples, as considered here. A solution for combining repeated measurement results and their individual uncertainties based on simple interval logic is proposed here. The individual measurement values and their uncertainties are compared with the calculated average value to see if this implies that another, possibly unknown, source of uncertainty is present. The model of the individual results is modified for this possible between-replicate effect so that the repeated measurements are consistent. Lack of consistency is a strong indication that the measurement is not fully under control and needs further development or investigation. This is not always possible, however and the method given here is proposed to ensure that the values of the repeated measurements agree with each other. A simple numerical example

is given showing how the method can be implemented in practice.

- Coefficient of contribution to the combined standard uncertainty (PDF)

*Authors: Rüdiger Kessel, Raghu Kacker and Michael Berglund*

The International Organization for Standardization (ISO) Guide to the Expression of Uncertainty in Measurement (GUM) describes a generic procedure for determining an estimate for the value of the measurand and its associated combined standard uncertainty from the estimates and their associated standard uncertainties for various input quantities. A user of the ISO-GUM who is interested in understanding, managing or improving the measurement procedure needs the details, usually expressed as an uncertainty budget, on how the estimate for the value of the measurand and its associated combined standard uncertainty were calculated. In particular, a user may be interested in quantifying the degrees of contribution to the combined standard uncertainty from its components. When the measurement equation is a linear function of uncorrelated input variables, the contribution from a component is usually quantified by the product of the component of uncertainty and its sensitivity coefficient. This paper introduces a coefficient of contribution that is suitable for both uncorrelated and correlated input variables. The proposed coefficient of contribution is useful for a variety of measurement equations. Correlations between input variables can significantly alter the relative importance of the contributions to the combined standard uncertainty from its components.

- Evolution of modern approaches to express uncertainty in measurement (PDF)

*Authors: Raghu Kacker, Klaus-Dieter Sommer and Rüdiger Kessel*

An object of this paper is to discuss the logical development of the concept of uncertainty in measurement and the methods for its quantification from the classical error analysis to the modern approaches based on the Guide to the Expression of Uncertainty in Measurement (GUM). We review authoritative literature on error analysis and then discuss its limitations which motivated the experts from the International Committee for Weights and Measures (CIPM), the International Bureau of Weights and Measures (BIPM) and various national metrology institutes to develop specific recommendations which form the basis of the GUM. We discuss the new concepts introduced by the GUM and their merits and limitations. The limitations of the GUM led the BIPM Joint Committee on Guides in Metrology to develop an alternative approach—the draft Supplement 1 to the GUM (draft GUM-S1). We discuss the draft GUM-S1 and its merits and limitations. We hope this discussion will lead to a more effective use of the GUM and the draft GUM-S1 and stimulate investigations leading to further improvements in the methods to quantify uncertainty in measurement.